HYDROLOGICAL PROCEDURE NO. 10

STAGE-DISCHARGE CURVES

1976

JABATAN PENGAIRAN DAN SALIRAN
KEMENTERIAN PERTANIAN MALAYSIA

Hydrological Procedure No. 10

STAGE-DISCHARGE CURVES

1976

Bahagian Parit dan Taliair
Kementerian Pertanian Malaysia

ACKNOWLEDGEMENT

This procedure has been prepared by the Hydrology Branch of the Drainage and Irrigation Department, Malaysia, in association with the Engineering Export Association (ENEX) of New Zealand, Inc.

Price: $5

Published by and obtainable from:

Publications Unit, Ministry of Agriculture, Jalan Swettenham,
Kuala Lumpur

STAGE-DISCHARGE CURVES 1976

Contributors:

CORNELIS TOEBES

AND

YEOH GAN CHEANG

1976

KEMENTERIAN PERTANIAN MALAYSIA

CONTENTS

1. INTRODUCTION

1.1 Objective

The objective of the procedure is to outline the techniques for the development of stage-discharge curves and stage-discharge rating tables and to describe rules for the presentation of curves and tables.

1.2 Purpose

Since a continuous measurement of discharge is not possible in open channels, a relation has to be developed between the stage and the discharge. Such a relation is usually called the "rating" and when plotted provides a curve, called the stage-discharge curve or rating curve. The rating curve may be converted into a tabular form and is then referred to as the stage-discharge rating table. A rating table is a more convenient method than a curve for converting stage values to discharge values.

1.3 Methodology

The stage-discharge curve is developed by carrying out discharge measurements over the range of stages encountered at a given cross section and to observe the water level with each discharge measurement.

The stage-discharge relation must be developed for the cross section on which the water level recorder (or stick gauge) is established. This cross section is referred to as the master cross section.

Discharge measurements or gaugings are normally carried out at the master cross section. If gaugings are carried out just below or just above the master cross section, it is assumed that no differences exist between the master cross section and the gauging cross section.

If gaugings are carried out some distance away from the master cross section, the gauging cross section is likely to differ from the master cross section and may have separate stick gauges. Such a cross section is referred to as the "gauging cross section" and water levels observed at such a gauging cross section are only used for correction purposes as explained in para. 5.2 and not for the development of the stage-discharge curve.

The stage-area relation is developed by surveying the master cross section accurately. The resulting curve is called the master stage-area curve.

The stage-mean velocity relation is developed at the master cross section also by calculating the mean velocity for each discharge measurement. The resulting curve is called the stage-mean velocity curve.

Both curves are required for extending the stage-discharge curve beyond the range of observations made and to assist in determining changes that may occur in the stage-discharge relation.

The "gauging cross section" may have bed level changes that differ from those at the "master cross section" and in this case a stage-area relation for the "gauging cross section" is developed also to allow corrections to be made as explained in para. 5.2.

2. CONSTRUCTION OF STAGE-DISCHARGE CURVES

2.1 Plotting

The stage-discharge curve is plotted on one graph sheet as shown in Fig. 1.

With the stage (in reduced level) fixed as the vertical axis (y-axis) and the discharge, the cross-sectional area and the mean velocity fixed as the horizontal axis (x-axis), plot in black ink the results of the gauging observations as illustrated in Fig. 1.

Choose linear scales which are convenient as well as easy for interpolation. For instance, for the horizontal scale, choose 1 cm = 20 m^3/s, 1 cm = 50 m^3/s etc.

No hard and fast rules can be given for plotting, but curves should show an adequate curvature to provide the sensitivity required for reading of changes in discharge with changes in stage.

Frequently, it is impossible to plot a curve which is sensitive over its entire range and for this reason low flow discharges are re-plotted to a more sensitive scale in an "inset" as specified in para 2.4. *See* inset in Fig. 1.

Plot the stage-discharge points by using the symbol " ʘ " (a circle and a dot). Adjacent to the plotted points label clearly the gauging numbers.

Having plotted the gauging points, draw provisional mean lines in pencil with the aid of an adjustable curve (e.g. a plastic flexible curve).

For drawing the curves, it may be assumed that the shape of the stage-discharge curve is approximately parabolic (*see* para. 3.3 for exceptions).

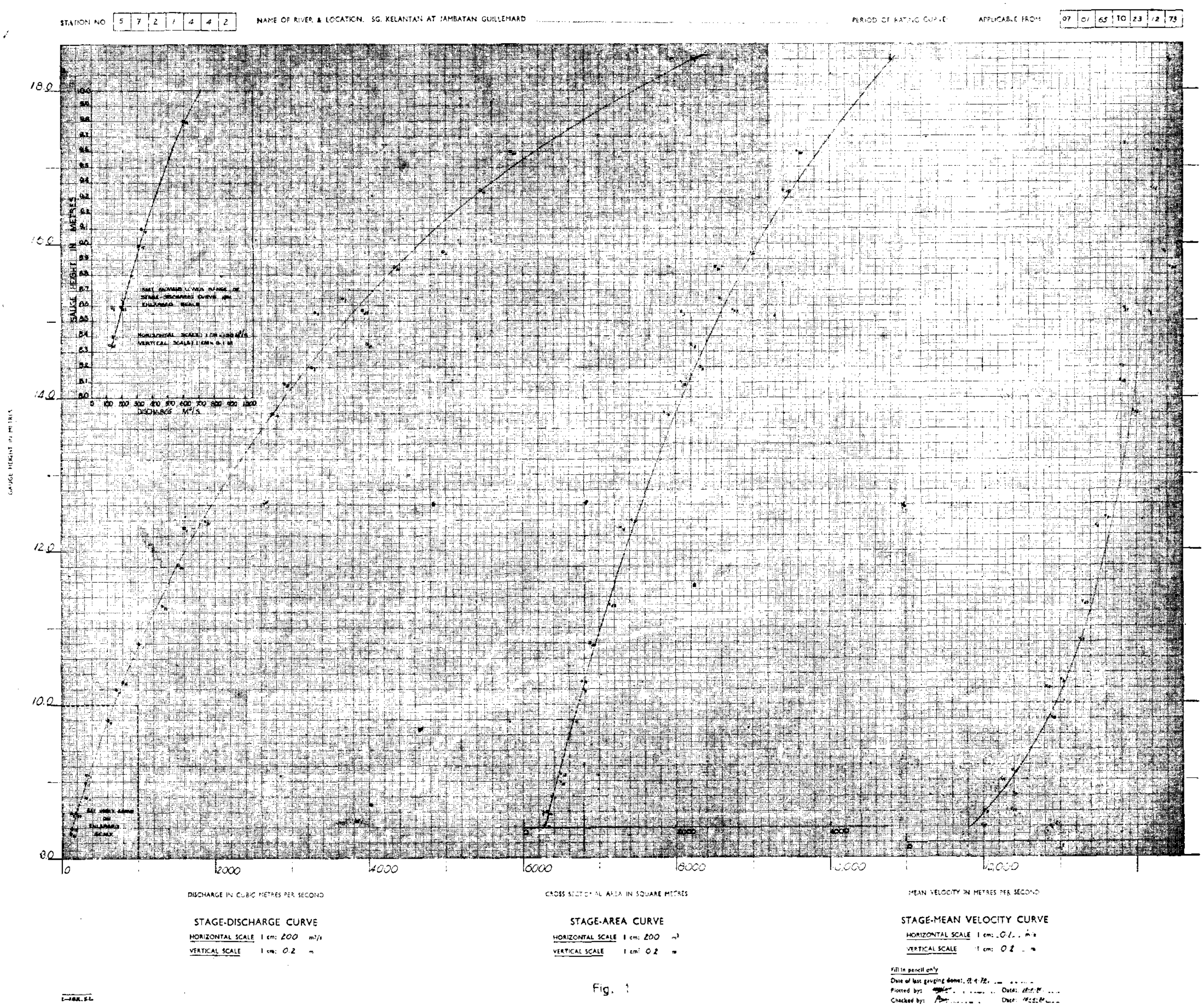
The stage-mean velocity curve is usually asymptotic to the vertical at the higher stages.

The master stage-area curve is drawn by connecting with a smooth line *all the points obtained from areas calculated from the master cross section.*

2.2 Fit of points on curves

In general, the points plotted form a dispersed array around the provisional mean curves. It is essential that each flow measurement deviating from the general trend be investigated to find the reason for its departure—this could be:

(*a*) mistake in booking and/or calculation of the discharge;



(Reduced from original size 73 cm × 53 cm)

(*b*) inaccurate discharge determination, e.g. a slope area observation; an insufficient number of velocity observations in a current meter gauging; current meter in poor working order while gauging etc.;

(*c*) inaccurate gauge height observation, e.g. the water level was not recorded and checked at the time of observation, or the zero gauge height has changed because the gauge has been shifted;

(*d*) non-steady flow conditions during the discharge observations—the rate of rise or fall should always be recorded; if rates of rise or fall are significant a loop rating curve may have to be constructed—note that gaugings made on the rising stage should fall on the right of the steady flow rating curve and falling stages on the left-hand side (refer to pages 5 to 8, Chapter 8, "Applied Hydrology" Vol. 2, by C. Toebes). It is important to realize that strictly speaking each flood would have its own "loop" and it is not correct to draw a loop rating curve based on gaugings carried out over a number of floods;

(*e*) backwater conditions during the gauging; this includes tidal effects, in such a case, a slope observation should be carried out simultaneously because the slope is required for a correction of the discharge. (refer to pages 8 to 11 Chapter 8, "Applied Hydrology" Vol. 2 by C. Toebes);

(*f*) changes in the cross section due to aggradation or degradation will affect the gauge height and in some cases the discharge. They can be detected by a study of the stage-area curve (*see* section 5);

(*g*) changes due to a change in weed growth etc. and consequent change in the roughness of the channel; a large number of gaugings should be carried out in these cases to detect the seasonal influences (*see* paragraph 3.3);

(*h*) inaccurate determination of the mean line.

After deleting all suspect gaugings, a final curve is drawn by hand, taking into account only those gaugings which deviate by no more than $\pm$ 4% from the provisional curve.

2.3 Period applicable to stage-discharge curves

The curves will only be valid if the controls in the reach governing the master cross section do not change. If changes occur, a new set of curves will have to be developed. It is important to label clearly the starting date and hour and finishing date and hour for which the old curves are applicable. Details as to how to determine changes in controls are given in para. 3.3.

2.4 Stage-discharge curve inset

The inset accommodates the portion of the stage-discharge curve which is plotted to a more sensitive scale. Consequently, the lower stage-discharge values should be read from the curve demarcated by the inset. It is preferable that an inset is drawn only when a sufficient number of low flow gaugings has been carried out.

3. EQUATION OF STAGE-DISCHARGE CURVES

3.1 Equation

If the flow in a reach is uniform (cross sections are approximately similar in shape and area in the reach) the general equation of the rating curve is parabolic i.e.

$$Q = K \ (H\text{-}a)^n$$

where Q is the discharge in cubic metres per second;
K is a constant,
H is the gauge height;
a is the gauge height for zero flow
and n is an exponent.

If the width/depth ratio is relatively large, the exponent
$n \triangleq$ 3/2 for a rectangular section;
$n \triangleq$ 2 for the concave section of a parabolic shape;
$n \triangleq$ 5/2 for a triangle and semi-circular section.

In irregular sections the exponent n varies. If the flow is non-uniform the general equation does not apply. Almost all natural channels have non-uniform flow but frequently may be assumed to approximate uniform flow. For more information on this aspect, refer to pages 1 and 2 Chapter 8, Vol. 2 "Applied Hydrology" by C. Toebes.

3.2 Logarithmic plotting

If a parabolic equation applies, the rating curve plotted on log-log paper will result in a straight line. Taking the logarithm of both sides of the general equation, we obtain:

$\log Q = \log K + n \log (H\text{-}a)$
which is equivalent to the equation of a straight line $y = mx + c$
where $y = \log Q$; $c = \log K$, the intercept and $x = \log (H\text{-}a)$.

Clearly we will have to know the value of "a" in order to solve the equation and to be able to plot the results.

Normally the plotting is done on logarithmic paper and this is convenient for constructing rating curves when but few gaugings are available.

It is not correct to pre-select a convenient exponent (say 2) in the general equation. In some cases, such an approximation may give a reasonable fit for the range of discharges considered but may be most misleading for discharges outside this range.

Logarithmic plotting cannot replace plotting of stage-discharge curves on a natural scale since the nature of the logarithmic values of numbers is such that points will lie very closely together at the higher stages, giving a false sense of accuracy.

Also the transfer of data from a "logarithmic" rating curve to a rating table may lead to serious errors because of this.

Logarithmic plotting is an excellent aid however when but few gaugings are available to determine the trend of the line, and is also of great value in determining to what stage controls are effective.

3.3 Breaks in stage-discharge curve

Sometimes flows become distinctly non-uniform at higher stages and the general equation does not, strictly speaking, apply. In such cases, the equation applies only for the lower stages; in other cases it applies throughout the range but with a change of the exponent n. Such cases are:

(*a*) where bank overflow occurs, there must be a break in the rating curve at the bank full stage level. The general equation may apply throughout the range of stage but the value of n changes at the bank full stage level (*see* Fig. 2).

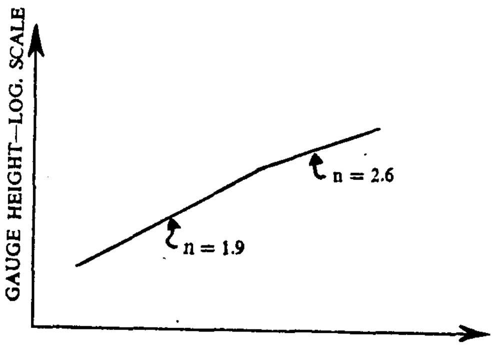


FIG. 2

(*b*) breaks in the rating curve also occur where controls become effective e.g. a constriction in the channel downstream of the gauging site may serve as a control from medium to high stages only; during these stages the equation may not apply, or if applicable, have a changed value of n (*see* Fig. 3);

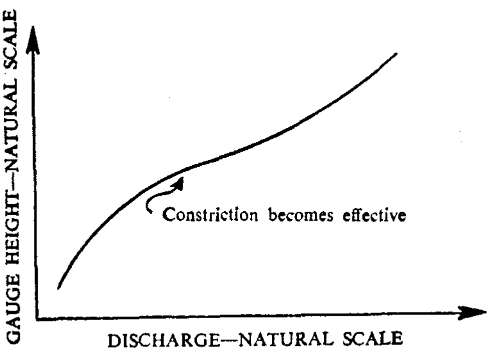


FIG. 3

(*c*) when there is a shifting control, the stage-discharge relation is not permanent but varies from time to time, either gradually or abruptly, owing to aggradation or degradation. Normally only the lower part of the curve would be significantly affected and a family of curves is sometimes constructed (*see* Section 5);

(*d*) changes in vegetation may affect the stage-discharge relation causing variations in the exponent n or in the equation. (*see* paragraph 2.2).

3.4 Gauge height for zero flow

Ordinarily "a" the difference between zero gauge height and the level of zero flow is not zero and its magnitude is unknown.

Plotting Q against H on log-log paper when "a" is not zero will produce a curve. If the value of "a" is known, Q can be plotted against (H-a) and a straight line results.

There are three methods of determining the value of "a":

(*a*) A field investigation is made of the gauging-station; normally the lowest level of the low flow control can be taken as the gauge height for zero flow.

(*b*) Various values of "a" are assumed until values of Q against (H-a) gives a straight line on log-log paper. Note that to obtain the best result, Q and (H-a) values should be selected from the average curve and not be actual values;

(*c*) (i) Assuming that the lower part of the rating curve is a parabola, three values of discharge u, v and w are selected from the known portion of this curve in such a way that the quantities form a geometric series e.g. 50-150-450. (Fig. 4).

(ii) Through u and v, vertical lines and through v and w horizontal lines are drawn to intersect at s and t; through v and u, a line is drawn that will intersect a line drawn through t and s at r, the latter point being at the elevation of the gauge height for zero flow. (Fig. 4).

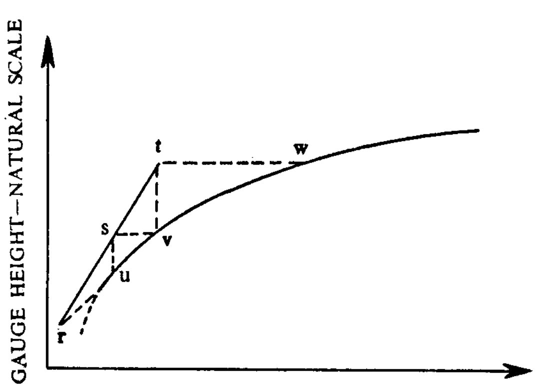


FIG. 4

In appendix A this method is given in a numerical form. *See* page. 13.

4. EXTENSION OF RATING CURVES

Extension of the stage-mean velocity and stage-area curves will, in general, give the most reliable results. The rate of increase of the mean velocity curve, plotted as discussed in para. 2.1 from calculated velocities, tends to decrease at the higher stages at sites where the cross section is fairly regular and no bank overflow occurs; in this case, the stage-mean velocity curve may be extended, considering in particular that the stage-mean velocity curve tends to be asymptotic at the higher stages.

A second method is the extension of the rating curve plotted on log-log paper. As discussed in para 3.2 this can only be carried out for those rating curves which fit the general equation. Curves should not be extended from low to high flow if bank overflow occurs, but first drawn to bank full stage and from there on to the highest stages (*see* para 3.3). The same applies when controls are effective for certain stages only (*see* para 3.3).

A third method applies only to sections without bank overflow. At the higher stages the factor $S^{\frac{1}{2}}/N$ in Manning's formula $V = \frac{R^{\frac{2}{3}}\ S^{\frac{1}{2}}}{N}$ (metric units) becomes approximately constant.

This formula can thus be rewritten $V = CR^{\frac{2}{3}}$.

By taking various values of V from the known portion of the mean velocity curve and the corresponding values of R, values of C can be computed for the range in stage for which the mean velocity curve is known.

By plotting these values of C against the gauge height, a curve is obtained that should tend asymptotically to the vertical. This line can thus be extended and values of C read off and combined with their respective values of $R^{\frac{2}{3}}$ and A.

5. CORRECTION FOR SHIFTING CONTROLS

5.1 Recorder section used as gauging section

Since a stable rating curve is only possible if there is a stable control, a shift in the control, or the absence of one, causes the rating curve to vary from the original one. Changes in the control (and in the master cross-section as a consequence) can be noted by plotting the cross-sectional area obtained with each gauging on the master stage-area curve. If two or more areas obtained with gaugings deviate from the curve on the same side of the curve, then a change in the cross section appears likely. Three different cases of shifting controls can be identified. In the first two cases it will be necessary to resurvey the master cross-section.

(*a*) if a sudden change in the bed has occurred after a big flood, or the low water control e.g., a rapid has moved downstream, the river has to be regauged and a new rating curve constructed. Usually, it is sufficient to reconstruct the lower part of the rating curve as higher stages will not be seriously affected;

(*b*) if degradation and aggradation of the bed at the gauging station occur in a regular fashion and the bed returns to a mean cross section in stable periods (e.g. floods causing a degradation on the rising hydrograph) it is best to construct a family of curves (*see* Fig. 5).

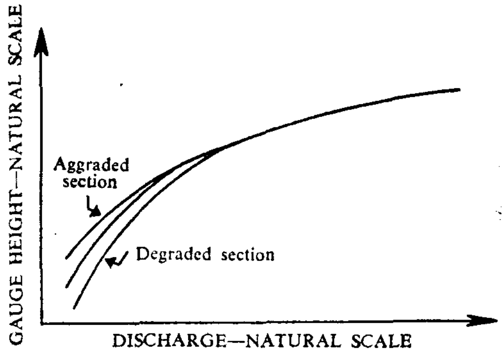


FIG. 5

(*c*) If the bed movement is irregular, a mean stage-area curve, i.e. the line of best fit, should be drawn through all area observations. For any further gauging, the discharge value is the product of the corrected area obtained from the mean stage-area curve and the mean velocity at that corresponding gauge height. It is to be noted that if a change would occur in the low and high water controls simultaneously, then it is still highly unlikely that the new rating curve would run parallel to the old curve in its entire length. If gaugings indicate that two or more parallel stage-discharge curves should be drawn, then most likely a change in the zero gauge height has occurred and one should investigate immediately whether gauges have been shifted without a proper datum correction. If bed changes occur frequently, serious consideration should be given to either constructing a low flow control, or relocating the station.

5.2 Gauging section some distance away from recorder section

As discussed in paragraph 1.3 it is assumed that any changes in the recorder cross section will be different from those in the gauging cross section. In this case, plot the cross-sectional areas obtained with each gauging on the gauging stage-area curve. If two or more areas obtained with gaugings deviate from the curve on the same side of the curve, then the master cross section need to be resurveyed immediately to check whether a change in the control has occurred.

6. APPROXIMATE METHODS

Sometimes a rating curve is required before any current meter gaugings are made or only an approximate curve (e.g. for flood warning purposes) is necessary. A provisional curve may be constructed by one of the following methods:

(*a*) by taking a series of float measurements and applying either a constant or varying factor to the surface velocity, a series of mean velocities is obtained. By constructing the stage-mean velocity curve and the stage-area curve, the provisional stage-discharge curve can be constructed.

(*b*) a field observation is made of the gauge height for zero flow (*see* para. 3.4) and an estimate of a flood discharge made by one of the following methods:

(i) if flood marks are available, a slope-area calculation is made;

(ii) if only an indication of the flood height is available then this flood is estimated using Lacey's formula $W = x\sqrt{Q}$, where W is the flood width and x is estimated (approx. 3.5 for very wide channels to 1.5 for very narrow and deep channels). An assumption is now made that the flow is uniform and the zero flow gauge height connected by a straight line with the flood discharge on log-log paper. (N.B. if bank overflow occurs, 2 curves will have to be constructed, one for below and one for over bank full stage—*see* para. 3.3). Note that a study of the cross-section and an estimation of the exponent "n" in the general equation may aid considerably in the first estimate of the rating curve.

7. RATING TABLES

For the conversion of stage into the corresponding discharge, a rating curve has to be converted into a rating table to allow a constant discharge to be read off at any gauge height. Rating tables will be derived from the stage-discharge curves in Hydrology Branch of D.I.D. Headquarters, Kuala Lumpur but for completeness sake, an example is given below of the derivation of a rating table. It is to be noted that because of a slightly different approach used for the digitizer*, slight differences may occur between manually calculated and digitized values.

A rating table is constructed by reading off the stage-discharge curve the discharge pertaining to each centimetre up to 5 m; for greater ranges a large interval is used. Then the first differences and the second differences are computed as shown in the Table I and II.

TABLE I (hypothetical example)

DRAINAGE AND IRRIGATION DEPARTMENT
MALAYSIA

STAGE-DISCHARGE RATING TABLE FOR SUNGEI SWETTENHAM No. 3116427 AT JALAN SWETTENHAM BRIDGE, No. 10/73 KUALA LUMPUR, FOR THE PERIOD 24-10-68 TO 21-11-73

THIS RATING TABLE IS BASED ON 41 CURRENT METER GAUGINGS AND TWO SURFACE FLOAT MEASUREMENTS

R.L.	*Discharge*	*1st. Diff.*	*2nd. Diff.*
11000	9150		
		180	
11010	9330		60
		120	
11020	9450		30
		150	
11030	9600		75
		225	
11040	9825		75
		150	
11050	9975		0
		150	
11060	10125		75
		225	
11070	10350		75
		150	
11080	10500		0
		150	
11090	10650		0
		150	
11100	10800		0
		150	
11110	10950		75
		75	
11120	11025		150
		225	
11130	11250		0
		225	
11140	11475		75
		150	
11150	11625		0
		150	
11160	11775		0
		150	
11170	11925		75
		225	
11180	12150		

* An electronic x-y coordinate machine used to trace and convert stage-discharge curves into stage-discharge values in the magnetic tape at the Hydrology Branch, D.I.D. Headquarters, Kuala Lumpur.

The table is now smoothed by a study of the 1st and 2nd differences. For a curve of ordinary shape the 1st and 2nd differences tend to increase gradually from low stage to high stage. Where rating curves approach a straight line at the higher stages, the second differences approach to zero as a limit.

Firstly, underline those discharge on the table which can be read off with absolute certainty e.g. at the junction of the graph paper ordinates; preferably such a constant discharge is selected about every ten discharges. Note that fixed points should at least be chosen at breaks in the rating curve. Secondly smooth the differences for each group of discharges and subsequently adjust the discharges as shown below Table II.

TABLE II

R.L.	*Discharge*	*1st. Diff.*	*2nd. Diff.*
11000	9150		
		150	
11019	9300		0
		150	
11020	9450		0
		150	
11030	9600		0
		150	
11040	9750		0
		150	
11050	9900		10
		160	
11060	10060		0
		160	
11070	10220		0
		160	
11080	10380		0
		160	
11090	10540		10
		170	
11100	10710		0
		170	
11110	10880		0
		170	
11120	11050		10
		180	
11130	11230		0
		180	
11140	11410		0
		180	
11150	11590		0
		180	
11160	11770		10
		190	
11170	11960		0
		190	
11180	12150		

Where logarithmic plotting is used, an equation of the curve can be derived and a rating table computed.

8. NOTIFICATION OF CHANGES

The Hydrology Branch, D.I.D. Headquarters, must be notified immediately of all changes or suspect changes in the discharge rating curve. This notification must include the date and time of possible change. Headquarters should also be supplied with stage-discharge, stage-area and stage-velocity curves for each gauging station whenever a change occurs, or at least once a year.

9. RANGE OF STAGE-DISCHARGE CURVES

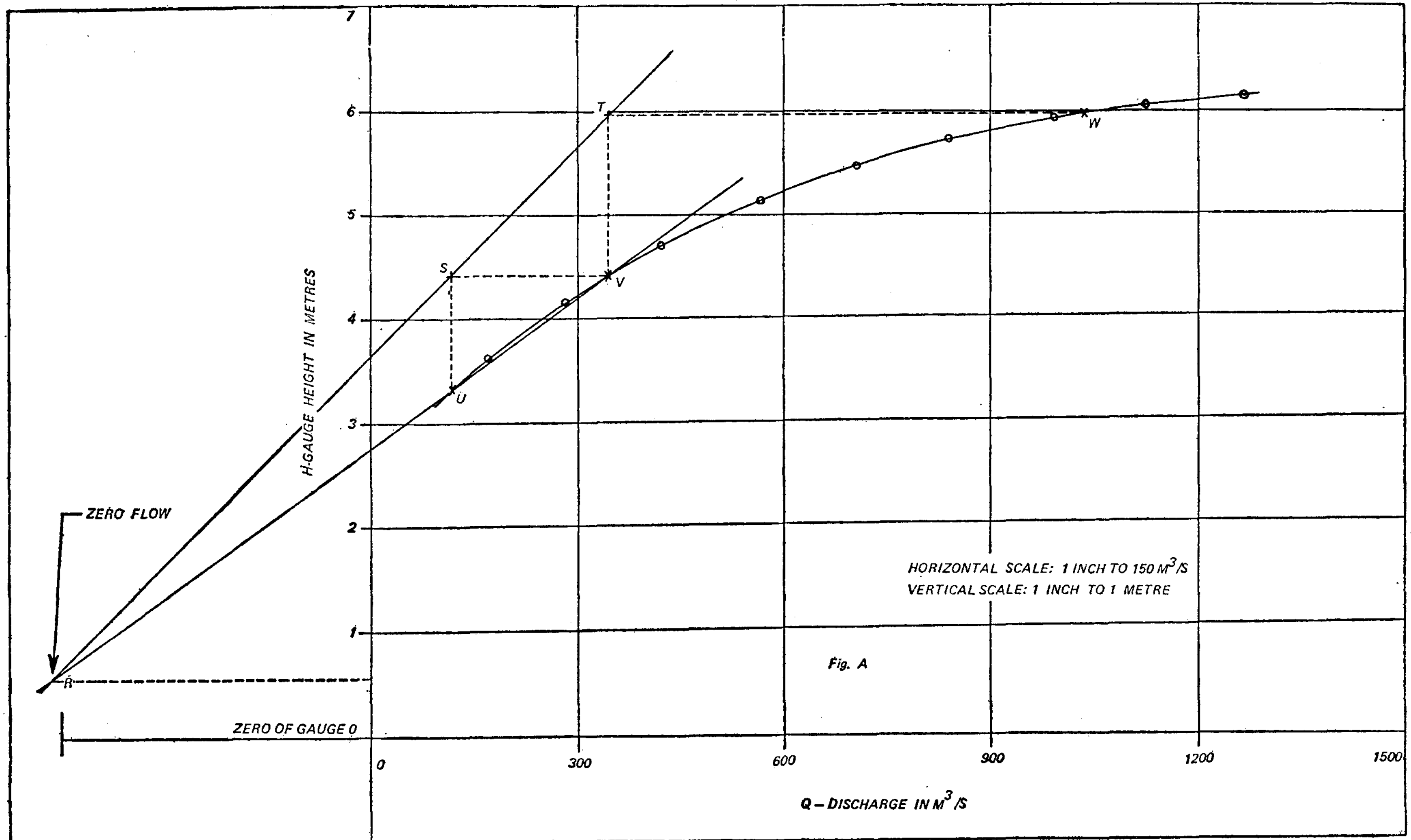
Since all data are processed by computer at D.I.D. Headquarters, it is essential that any stage-discharge curve submitted should cover the range of flow that can ever be acquired at the station. The curves should be extended approximately 30 cm below the lowest possible flow and 60 cm above the highest possible flow that could occur. These extensions should be given in dotted lines.

Appendix A

DETERMINATION OF GAUGE HEIGHT CORRESPONDING TO ZERO FLOW

In Fig. A, the abscissae of U, V and W are 115, 345 and 1035 (m^3/ s) respectively. Vertical and horizontal lines, through U, V and W, are drawn to intersect at S and T. Two straight lines TS and VU are drawn, both produced, to meet at R. This point is then the gauge height corresponding to zero flow (or zero discharge). In this case, the zero flow is located at +0.57 of the gauge height.

Fig. A on page 14.



H-GAUGE HEIGHT IN METRES
Q – DISCHARGE IN M³/S
ZERO FLOW
ZERO OF GAUGE 0
HORIZONTAL SCALE: 1 INCH TO 150 M³/S
VERTICAL SCALE: 1 INCH TO 1 METRE
R
S
T
U
V
W
0
300
600
900
1200
1500
1
2
3
4
5
6
7

Fig. A

EXTENSION OF STAGE-DISCHARGE CURVE

To construct an extension of the stage-discharge curve on logarithmic paper, it is imperative to convert gauge height (H) readings for the corresponding discharge (Q), which are taken from the stage-discharge curve, into gauge height readings reckoned from the level of zero flow.

In Fig. A, Appendix A, the zero flow is located at +0.57 metre of the gauge height. As the zero of gauge is 0.57 metre below the level of zero flow, 0.57 metre must be substracted from all gauge height readings (H).

The table below gives a set of readings taken from the stage-discharge curve, Fig. A, in Appendix A, and (H-a) values. (a = 0.57m which is the gauge height for zero flow).

H Gauge Height (m)		H-a (m)		Q Discharge m^3/s
3.32	...	2.75	...	115
4.42	...	3.85	...	345
4.71	...	4.14	...	424.5
5.50	...	4.93	...	707.5
5.96	...	5.39	...	1035

Values of (H-a) are then plotted against values of Q on logarithmic paper, Fig. B. The points plotted lie almost on a straight line. (If it does not, a further construction for zero flow is repeated by using other points on the discharge rating curve).

See page 16.

EQUATION: $Q = K\ (H\text{-}a)^n$

As mentioned in para 3.1, the general equation of the rating curve is:

$$Q = K\ (H\text{-}a)^n$$

from which, taking logarithm to base 10,

$$\log Q = \log K + n \log (H\text{-}a)$$

The plotted points, Fig. B, lie very nearly on a straight line.

From the graph, when $Q = 115$, $(H\text{-}a) = 2.75$
and when $Q = 1035$, $(H\text{-}a) = 5.39$

Substituting these values in the equation above, we obtain two simultaneous equations:

$$\log 115 = \log K + n \log 2.75$$
$$\log 1035 = \log K + n \log 5.39$$

On solving, $n = 3.26$ and $K = 4.23$ therefore the required equation is $Q = 4.23\ (H\text{-}0.57)^{3.26}$

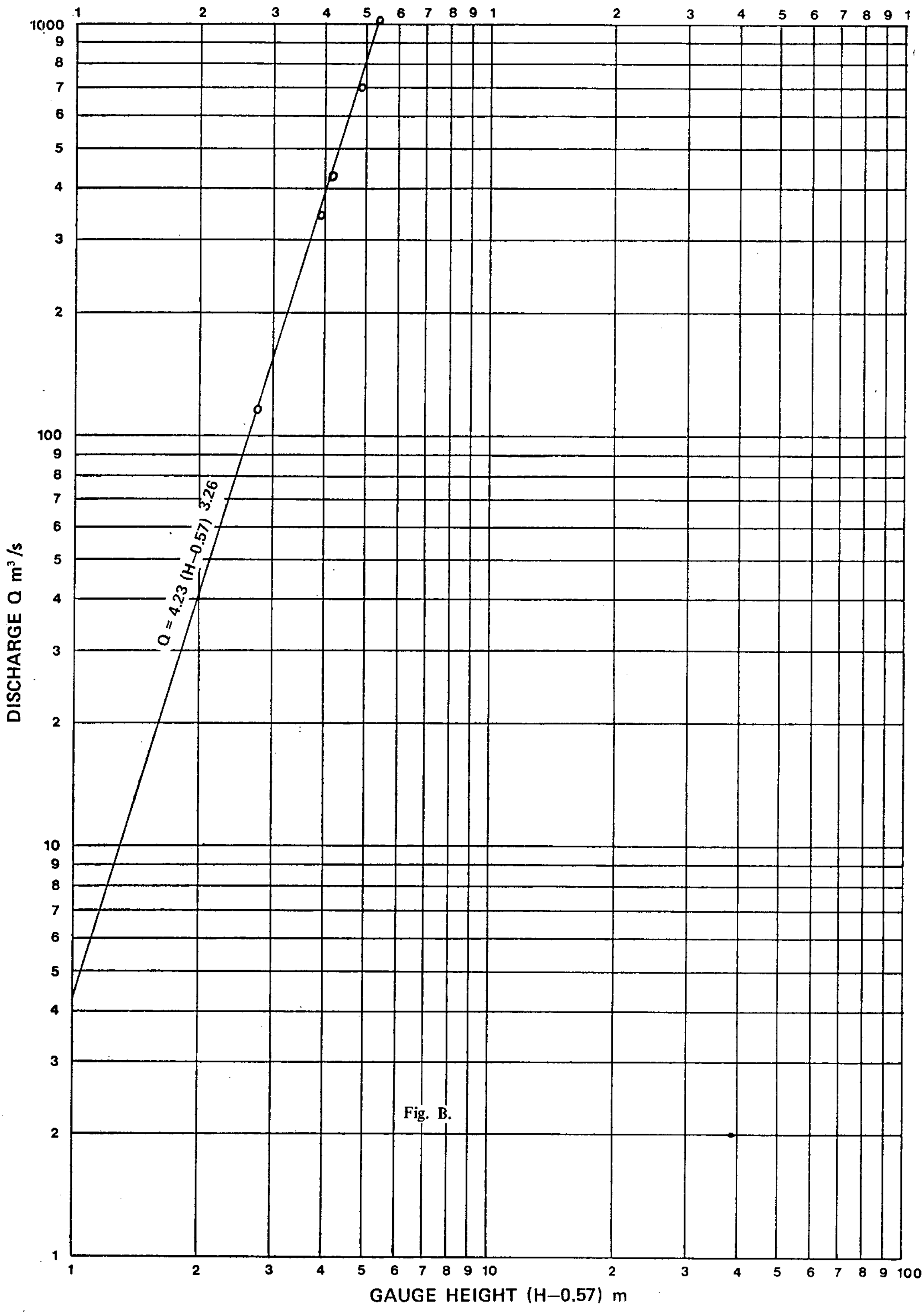


Fig. B.

The values of n and K can be determined also graphically from Fig. B. The intercept of the straight line on the vertical axis gives K = 4.23, while the slope (tangent of the angle which the straight line makes with the horizontal axis) gives n = 3.26.

It is common practice to fix the gauge height on the vertical axis and the discharge on the horizontal axis as in Fig. C.

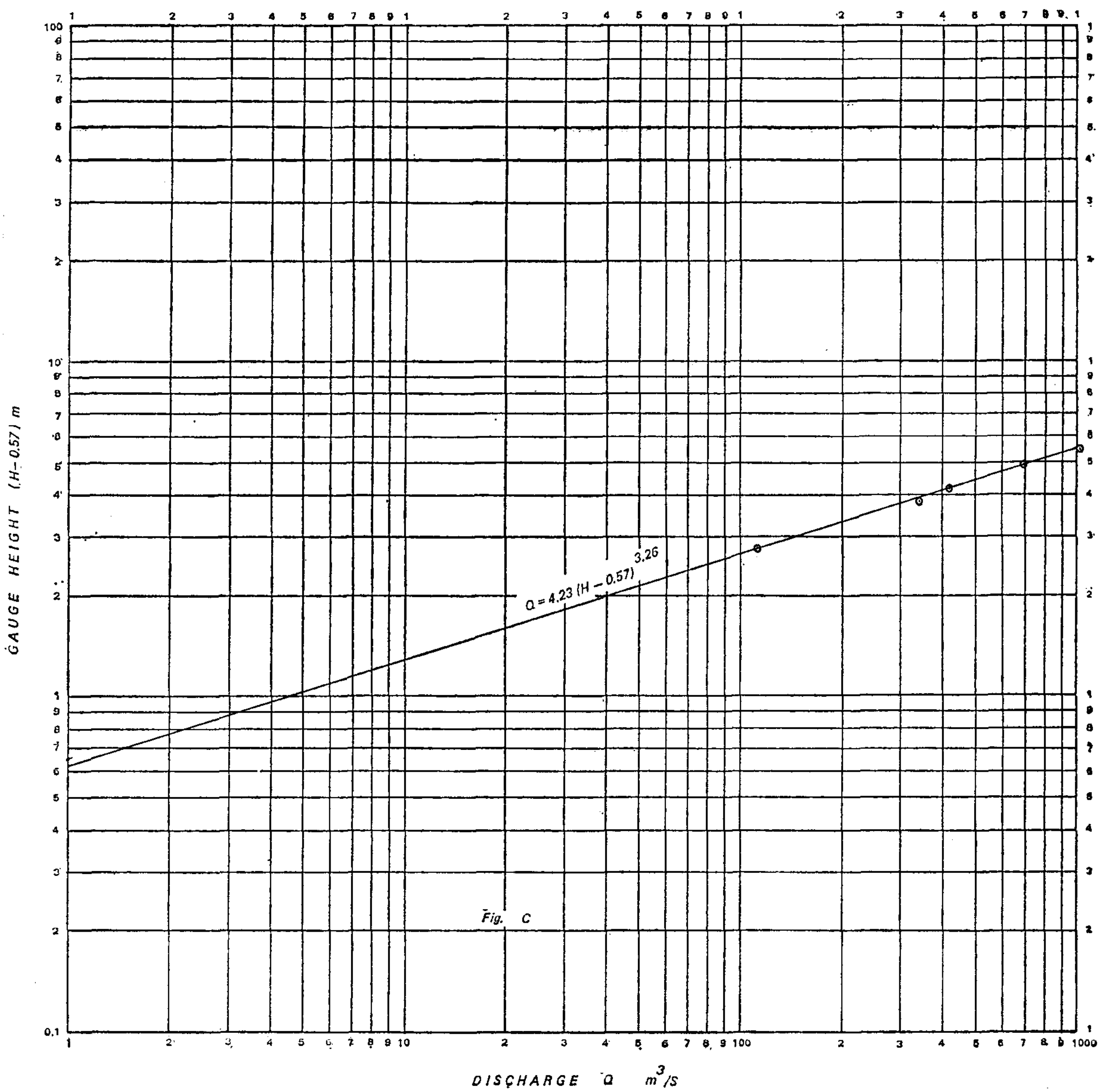


Fig. C

PROCEDURES PREVIOUSLY PUBLISHED

		Price
1.	Estimation of the Design Rainstorm (1973)	\$8.00
2.	Water Quality Sampling for Surface Water (1973)	\$3.00
3.	A General Purpose Event Water Level Recorder (1973)	\$5.00
4.	Magnitude and Frequency of Floods in Peninsular Malaysia (1974)	\$6.00
5.	Rational Method of Flood Estimation for Rural Catchments in Peninsular Malaysia (1974)	\$3.00
6.	Hydrological Station Numbering System (1974)	\$3.00
7.	Hydrological Station Registers (1974)	\$5.00
8.	Field Installation and Maintenance of Capricorder 1599 (1974) ...	\$5.00
9.	Field Installation and Maintenance of Capricorder 1958 Digital Event Water Level Recorder (1974)	\$5.00

60903—6-1-77.

DICETAK OLEH
IBRAHIM BIN JOHARI, P.I.S., KETUA PENGARAH PERCETAKAN
SEMENANJUNG MALAYSIA, KUALA LUMPUR
1977